

Electric charge in the stochastic electric field

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The influence of electric stochastic fields on the relativistic charged particles is investigated in the gauge invariant path integral formalism. Using the cumulant expansion one finds the exponential relaxation of the charge Green's function both for spinless and Dirac charges.

1.

Stochastic electromagnetic fields and specifically stochastic electric fields play important role in many areas of physics and technology, from extragalactic fields and cosmology [1, 2], to molecular physics [3] medicine (e.g. tomography), accelerator physics [4]; for a general theory see [5]. Very intensive electric and magnetic fields occur in the heavy ion collisions [6] with possible stochastic component.

The treatment of the dynamics of the system in a stochastic field can be done on the general grounds in the formalism of the (world-line) path integrals [7, 8], which is applicable to any system and in principle, to any field, see [9] for a recent development. In particular it was shown in [7, 10, 11], that stochastic colorelectric fields give rise to the basic phenomenon of confinement. In this derivation it was essential, that the correlation length λ of stochasticity is much smaller, than other ranges in the theory, e.g. the radius of orbital motion etc. Another important feature was the Euclidean character of colorelectric fields, which ensures the real confining potential.

In this paper we apply the path integral approach to the relativistic field theory in the external real electric fields, and discuss the simplest situation, when the correlation length λ (correlation time τ) is much smaller than other lengths (periods) in the problem, denoted by Λ , i.e. the free path (time interval) of the charge between collisions, the range of constant potential (period of orbital motion) on the atomic or molecular level.

In this case the effect of stochasticity enters as an additional term in the total Hamiltonian; it is imaginary for stochastic electric fields. In the opposite case, $\lambda > \Lambda$, the stochastic fields are acting on the defined motion of the particle, and the formalism becomes more complicated.

2.

We start with the Green's function of one spinless charged relativistic particle in the external electromagnetic field, which will be treated as stochastic in the cumulant formalism [12]. To this end we write down the path integral form for the Green's function, given by the Fock–Feynman–Schwinger (FFS) path integral [7, 8]

$$G(x, y) = (m^2 - D_\mu^2)_{xy}^{-1} = \int_0^\infty ds D^4 z e^{-K} \Phi(x, y), \quad (1)$$

where

$$\Phi(x, y) = \exp(i e \int_y^x A_\mu(z) dz_\mu), \quad (2)$$

$$K = \int_0^s [m^2 + \frac{1}{4} \left(\frac{dz_\mu}{d\tau} \right)^2] d\tau. \quad (3)$$

$A_\mu(z)$ can include both stochastic and nonstochastic (e.g. Coulombic) components, but we shall start for simplicity with the stochastic part only. It is convenient to work out the final expressions in the Euclidean space-time, and then go over to the Minkovskian space-time, however finally, all time fluctuations and the fields should considered in the real (Minkovskian) time.

Choosing the virtual energy ω instead of the proper time s as a variable, $s = \frac{T}{2\omega}$, $T = x_4 - y_4$, one has

$$\int_0^\infty ds (D^4 z) \Phi(x, y) e^{-K} = T \int_0^\infty \frac{d\omega}{2\omega^2} (D^3 z)_{\mathbf{xy}} e^{-K(\omega)} \Phi(x, y) \quad (4)$$

where

$$K(\omega) = \int_0^T dt_E \left(\frac{\omega}{2} + \frac{m^2}{2\omega} + \frac{\omega}{2} \left(\frac{dz}{dt_E} \right)^2 \right) \quad (5)$$

To define stochastic fields in a gauge-invariant way as averages of field strengths one considers any trajectory among the integral paths and combine it with a fixed shadow trajectory, which is needed to complete

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the closed contour C . This helps to express the vector potential $A_\mu(z)$ in terms of the field strength, e.g. via the representation [13]

$$A_\mu(z) = \int_0^z F_{\nu\mu}(u) \alpha(u) du_\nu, \quad (6)$$

and $\Phi(x, y)$, which is now a part of the Wilson loop $C(x, y)$, including the charge trajectory in $\Phi(x, y)$ and the shadow trajectory, can be rewritten in the form

$$W(x, y) = \exp \left(\int_{C(x, y)} ie A_\mu(z) dz_\mu \right) = \exp \left(ie \int ds_{\mu\nu}(u) F_{\mu\nu}(u) \right), \quad (7)$$

where $ds_{\mu\nu}$ is the surface element in the area inside $C(x, y)$, $ds_{\mu 4} = du_\mu dz_4$, and dz_4 is the Euclidean time element $dz_4 = idz_0$,

At this point it is important to define the stochastic process [5, 12], namely, to take into account, that $\Phi(x, y) \rightarrow W(x, y)$ enters as an average value in the averaging procedure, which is characterized by the gauge invariant correlators of the field strengths, e.g.

$$e^2 \langle F_{\mu\nu}(x) F_{\lambda\sigma}(y) \rangle = \frac{1}{2} \left(\frac{\partial}{\partial w_\mu} (w_\lambda D^{(2)}(w)) \delta_{\nu\sigma} + \frac{\partial}{\partial w_\nu} (w_\sigma D^{(2)}(w)) \delta_{\mu\lambda} \right) \quad (8)$$

and similarly for the third and higher powers of $F_{\mu\nu}$. Here $w_\lambda = x_\lambda - y_\lambda$, and $D^{(2)}$ is a scalar function of its arguments, $D^{(2)}(w) \rightarrow D^{(2)}(w_i^2 + w_4^2) = D^{(2)}(w_i^2 - w_0^2)$.

One can now express the average value $\langle W \rangle$ in terms of cumulants of field correlators [9, 12], assuming that $\langle F_{\mu\nu} \rangle \equiv 0$,

$$\langle W(x, y) \rangle = \exp \sum \frac{(ie)^n}{n} \int ds_{\mu_1\nu_1}(1) \dots ds_{\mu_n\nu_n}(n) \langle \langle F_{\mu_1\nu_1}(1) \dots F_{\mu_n\nu_n}(n) \rangle \rangle \quad (9)$$

where $\langle \langle F \dots F \rangle \rangle$ are cumulants, defined as follows

$$\langle \langle F_{\mu\nu_1}(1) F_{\mu_2\nu_2}(2) \rangle \rangle \equiv \langle F_{\mu_1\nu_1}(1) F_{\mu_2\nu_2}(2) \rangle \quad (10)$$

$$\begin{aligned} \langle \langle F_{\mu_1\nu_1}(1) F_{\mu_2\nu_2}(2) F_{\mu_3\nu_3}(3) F_{\mu_4\nu_4}(4) \rangle \rangle = \\ \langle F(1) F(2) F(3) F(4) \rangle - \langle F(1) F(2) \rangle \langle F(3) F(4) \rangle - \\ \langle F(1) F(3) \rangle \langle F(2) F(4) \rangle - \langle F(1) F(4) \rangle \langle F(2) F(3) \rangle. \end{aligned} \quad (11)$$

see [12] for more details.

In principle, as it was shown in [9], one can express QED or QCD dynamics in terms of correlators

, e.g. for the Coulomb interaction (the field correlator of the photon exchange) the corresponding correlator is $D^{(Coul)}(w)$,

$$D^{(2)(Coul)}(w) = \frac{4\alpha}{\pi(w_i^2 + w_4^2)^2}, \quad (12)$$

and its contribution to $\langle W \rangle$ is

$$\langle W(x, y) \rangle = \exp \left(i \int \frac{e^2 dz_0}{r(z_0)} \right) \quad (13)$$

where $r(z_0)$ is the distance between the point $\mathbf{z}(z_0)$ on the trajectory of the particle, and the point $\mathbf{Z}(z_0)$ on the shadow trajectory, which can refer to the opposite charge companion of our particle. In the case when one can neglect the Coulomb interaction of the particle with all neighbors, the correlator (12) is absent, and we assume, that all stochastic correlators are fast decreasing, both in time and in space variables, so that the average $\langle W \rangle$ factorizes into a product of averages separately for a given charge trajectory and a shadow trajectory, which will not be of interest to us.

As a result the average $\langle W \rangle$ in (9) contains integrations $ds_{\mu_i\nu_i}$ in the vicinity of the trajectory $z_\mu(\tau) \rightarrow z_\mu(t)$.

Keeping only the term $n = 2$ in (9), one obtains in the exponent of (9)

$$\begin{aligned} \Gamma_E T_4 = \frac{e^2}{2} \int ds_{\mu_1\nu_1}(u) ds_{\mu_2\nu_2}(v) \langle F_{\mu_1\nu_1}(u) F_{\mu_2\nu_2}(v) \rangle = \\ \frac{1}{4} \int ds_{\mu_1\nu_1}(u) ds_{\mu_2\nu_2}(v) \left(\frac{\partial}{\partial w_{\mu_1}} (w_{\mu_2} D^{(2)}(w)) \delta_{\nu_1\nu_2} + \right. \\ \left. \frac{\partial}{\partial w_{\nu_1}} (w_{\nu_2} D^{(2)}(w)) \delta_{\mu_1\mu_2} \right). \end{aligned} \quad (14)$$

For the stochastic electric fields $F_{\mu_1 4}, F_{\mu_2 4} = iE_{\mu_1}, iE_{\mu_2}$ one obtains, assuming that the surface in the loop $C(x, y)$ lies in the x_1, x_4 plane, $0 \leq u_1, v_1 \leq R$; $0 \leq u_4, v_4 \leq T_4$

$$\Gamma_E = \frac{1}{2} \int_0^R du_1 \int_0^R dv_1 \int_{-T_4}^{T_4} dw_0 \left(\frac{\partial}{\partial w_1} (w_1 D^{(2)}) + \frac{\partial}{\partial w_0} (w_0 D^{(2)}) \right). \quad (15)$$

We assume, that $D^{(2)}$ falls off fast for large $|w_4| = |w_0|$, so that the last factor in (15) vanishes, while the first term yields for the charge trajectory (a similar answer results for the background one)

$$\Gamma_E = \int_{-T_0}^{T_0} dw_0 \int_0^R dx \cdot x D^{(2)}(x, w_0). \quad (16)$$

Note, that $D^{(2)}(0,0) \geq 0$ for real electric fields. As a result the leading quadratic correlator leads to the following result for $\langle W \rangle$

$$\langle W(x, y) \rangle = \exp(-\Gamma_E T_0), \quad T_0 = x_0 - y_0 > 0. \quad (17)$$

Note, that we have used in (15) both temporal and spacial stochastic correlations, present in the correlator $D^{(2)}$ in (16).

Inserting (17) into (4) in the place of $\Phi(x, y), \langle \Phi(x, y) \rangle \rightarrow \langle W(x, y) \rangle$, one obtains ($T_4 \equiv T = iT_0$)

$$\langle G(x, y) \rangle_A = \sqrt{\frac{T_4}{8\pi}} \int_0^\infty \frac{d\omega}{\omega^{3/2}} (D^3 z)_{\mathbf{x}\mathbf{y}} e^{-K(\omega) + i\Gamma_E T_4}. \quad (18)$$

Doing the $(D^3 z)$ integration as in [9], one introduces the Hamiltonian $H(\omega)$

$$\langle G(x, y) \rangle_A = \sqrt{\frac{T_4}{8\pi}} \int \frac{d\omega}{\omega^{3/2}} \langle \mathbf{x} | e^{-(H(\omega) - i\Gamma_E)T_4} | \mathbf{y} \rangle, \quad (19)$$

where $H(\omega)$ is,

$$H(\omega) = \frac{\mathbf{p}^2 + m^2}{2\omega} + \frac{\omega}{2} + V_Z(r) \quad (20)$$

and we have included the potential $V_Z(r)$ in the case, when our particle is inside an atom or a hadron.

Integrating over $d\omega$ by the stationary point method as in [9], one arrives at the final form,

$$\langle G_Z(x, y) \rangle = \frac{1}{2m} \sum_n \varphi_n^*(\mathbf{x}) \varepsilon^{-i\varepsilon_n T_0 - \Gamma_E T_0} \varphi_n(\mathbf{y}), \quad (21)$$

where $\varphi_n(x)$ is atomic bound wave function, while

$$\varepsilon_n = M_Z + m \sqrt{1 - \frac{(Z\alpha)^2}{n^2}}. \quad (22)$$

So far the spinless particles were considered. Now, for particles with the spin 1/2 one should add in the exponent of (7) the term (see [7, 8]),

$$ie \int \sigma_{\mu\nu} F_{\mu\nu} \frac{dt_4}{2\omega}; \quad \sigma_{\mu\nu} F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma} \mathbf{H} & i\boldsymbol{\sigma} \mathbf{E} \\ i\boldsymbol{\sigma} \mathbf{E} & \boldsymbol{\sigma} \mathbf{H} \end{pmatrix}. \quad (23)$$

Keeping only Gaussian (quadratic) correlator of $\mathbf{E}(x, t)$, and going to the Minkovskian time, one obtains in the exponent of (14) for the spinor case

$$\Gamma_E \rightarrow \Gamma_E + \Gamma_E^\sigma \hat{1}, \quad \Gamma_E^\sigma = \frac{1}{8m^2} \int \langle E_i(z_0) E_i(z'_0) \rangle d(z_0 - z'_0). \quad (24)$$

Here $\hat{1}$ is the unit 4×4 Dirac matrix. Thus one can see, that spin-dependent forces, bring about additional relaxation of the Green's function.

3.

We have shown above, that high-frequency stochastic electric fields with the correlation length (time) smaller than other periods of time in the particle motion, create the relaxation of the particle signal. The relaxation widths $\Gamma_E, \Gamma_E^\sigma$ are proportional to the quadratic cumulant of the field strength. One can check, that higher correlators do not change qualitatively the situation. It is clear, that the weakening of the signal of the given total energy (momentum) is caused by the redistribution of energies (momenta) in the combined collective signal as can be seen by expanding fluctuating field $E_i(x, t)$ in the Fourier series, and this should be described in the framework of the general stochastic approach [2, 5]. One example where these processes may be important, is the behavior of the stochastic quark-gluon ensemble in the process of heavy ion collisions, where occur electric and magnetic fields of high intensity [6].

Consider now any macroscopic current $J(x, t)$, which is a statistical or coherent sum of elementary currents with the Green's functions given by Eq.(1). It is clear, that the correlator of the currents $\langle J(x, t) J(x', t') \rangle$ will be proportional to $\exp(-\Gamma_E |t - t'|)$, implying that the high-frequency stochastic background creates the universal relaxation of the signal, since Γ_E in (16) depends only on the stochastic source characteristics, namely on its high-frequency tail. It is remarkable from the general physical point that the same stochastic process with Euclidean colorelectric fields in QCD creates confinement and hence all our world, while in QED an analogous stochastic process simply yields a universal damping of any signal.

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